Fusion in the Sun

Towards the centre of the Sun, Hydrogen nuclei are converted into Helium nuclei by the process of nuclear fusion. (This is the mechanism involved in the explosive energy production in a Hydrogen bomb).

The Sun consists largely of Hydrogen nuclei (protons), represented by ^1_1H. At the high temperatures which prevail, electrons have been stripped from the Hydrogen atoms. These so-called "free" electrons make a contribution towards the internal pressure, balancing the gravitational forces. Thus the Sun is stable.

These Hydrogen nuclei $\left(^1_1H\right)$ repel one another because of the powerful, repelling, electrostatic forces between them. In fact,

$$\begin{pmatrix}\text{electrostatic force}\\ \text{between two protons}\end{pmatrix} = \sim 10^{40} \times \begin{pmatrix}\text{gravitational force}\\ \text{between two protons}\end{pmatrix}$$

Both obey the inverse-square law, so they have the same spatial dependence.

However, if they are moving quickly enough (that is, have enough kinetic energy $\left(= \frac{1}{2} m_p v_p^2\right)$, the repelling forces can be overcome. When this occurs, the Hydrogen nuclei can bind, leading to a more stable configuration, provided $T > 10^7$ K.

Particles and masses

Particle	Mass (kg)	
proton	$1{\cdot}673 \times 10^{-27}$	The mass difference, Δm, between four protons and one Helium nucleus = $4{\cdot}37 \times 10^{-29}$ kg
Helium nucleus	$6{\cdot}6465 \times 10^{-27}$	
positron, e^+	$9{\cdot}11 \times 10^{-31}$	
neutrino	negligible	

The overall reaction involves four protons fusing, to form a single Helium nucleus. Incidentally, in our Sun, the average "survival" time of a proton before it fuses is 10^{10} years (yes, 10^{10} years). However, there are lots of protons around, so the nuclear fusion processes are maintained.

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Two positrons and two neutrinos are also released, along with some energy. This energy is released in two forms: kinetic energy of the particles and gamma photons.

$$4\,{}^{1}H \longrightarrow {}^{4}He + 2e^{+} + 2\nu + \text{energy}$$

On the previous page, we saw that when a stable nucleus is formed from its constituent nucleons, the mass of the system decreases by $\Delta m = 4 \cdot 37 \times 10^{-29}$ kg.

Using (probably) the most famous scientific equation *($\Delta E = \Delta m . c^2$) and substituting:

$$\Delta E = 4 \cdot 37 \times 10^{-29} \text{ kg} \times (3 \times 10^{8} \text{ m s}^{-1})^2$$

$$= 3 \cdot 9(3) \times 10^{-12} \text{ kg m}^2 \text{ s}^{-2}$$

And $\Delta E = 3 \cdot 9(3) \times 10^{-12}$ J [Using my nuclear-powered slide rule]

This is a small amount of energy. However, a large number of nuclei is going through this reaction at the same time, so the amount of energy released every second becomes significant.

Measurements show that $L_{\odot} = 3 \cdot 83 \times 10^{26}$ W (J s^{-1}). From this, we can calculate the number of reactions occurring per second.

$$\left.\begin{matrix}\text{number of reactions} \\ \text{occurring per second}\end{matrix}\right\} = \frac{\text{Luminosity of the Sun}}{\text{energy released per reaction}}$$

$$= \frac{3 \cdot 83 \times 10^{26} \text{ J s}^{-1}}{3 \cdot 9(3) \times 10^{-12} \text{ J reaction}^{-1}}$$

$$= 9 \cdot 75 \times 10^{37} \text{ reactions s}^{-1}$$

Furthermore, (always a dreaded word), the mass of the Sun converted to energy per second $= \dfrac{dm}{dt}$

$$= \frac{3 \cdot 8(3) \times 10^{26} \text{ J s}^{-1}}{9 \times 10^{16} \text{ m}^2 \text{ s}^{-2}}$$

$$= 4 \cdot 3 \times 10^{9} \text{ kg s}^{-1}$$ [NPSR]

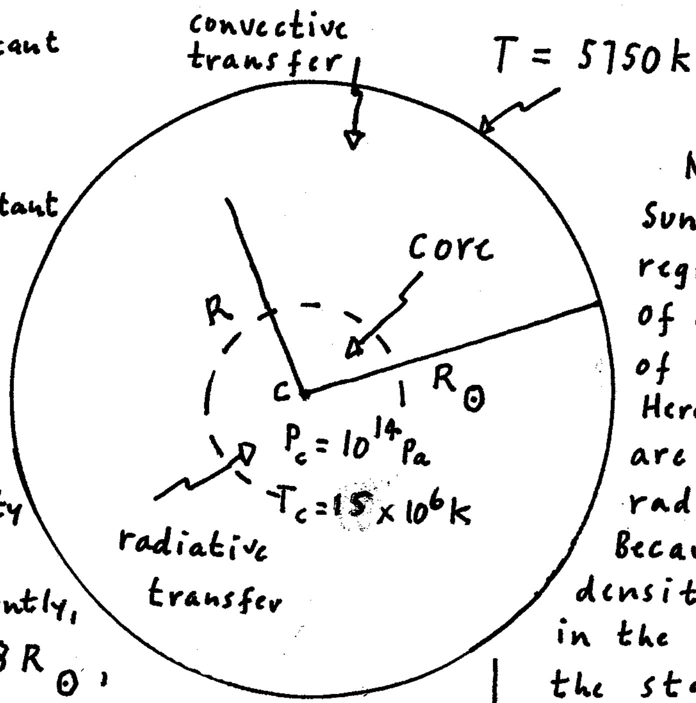
* Einstein's Equation

The Sun

$\frac{dT}{dR}$ is not constant

$\frac{dP}{dR}$ is not constant

The gas in outer parts of a star is cooler and this increases the opacity (the resistance to radiation). Consequently, beyond about $0 \cdot 8 R_{\odot}$, convection takes over. Rising currents of hot gas give rise to the granular appearance of the Solar surface.



Nuclear fusion in the Sun occurs in the central region, with a radius of about one-quarter of the whole star. Here, high-energy photons are produced, and they radiate outwards. Because of the high density of the gases in the innermost parts of the star, these photons are absorbed and re-emitted as a lower-energy photons on the route outwards. As a result, the whole process can take one million years.

Mass, Luminosity and Lifetime

As long as a protostar has a mass between $\sim 0 \cdot 08 M_{\odot}$ and $60 \rightarrow 100 M_{\odot}$, fusion will commence and a star will become a main-sequence star. The star will spend most of its life in this region of the Hertzsprung-Russell diagram.

The lifetime of a star in the main-sequence is linked to its luminosity and mass because:

(i) The greater its mass, the longer it will last before its supply of Hydrogen is consumed.

(ii) The greater its luminosity, the sooner the supply of Hydrogen will be exhausted.

When stars form from clouds of gas in space, their composition is initially $\sim 73\%$ Hydrogen, 25% Helium and 2% other elements, by mass. During their main-sequence lifetime, energy is produced by the proton-proton chain, until the Hydrogen reserve runs low. Our Sun is around 5×10^9 years old. It has more Hydrogen in its core; nevertheless, it still has enough Hydrogen to maintain its present level of energy production (thereby resulting in a loss of mass rate $\sim 4 \cdot 3 \times 10^9$ kg s^{-1}) for another five billion years.

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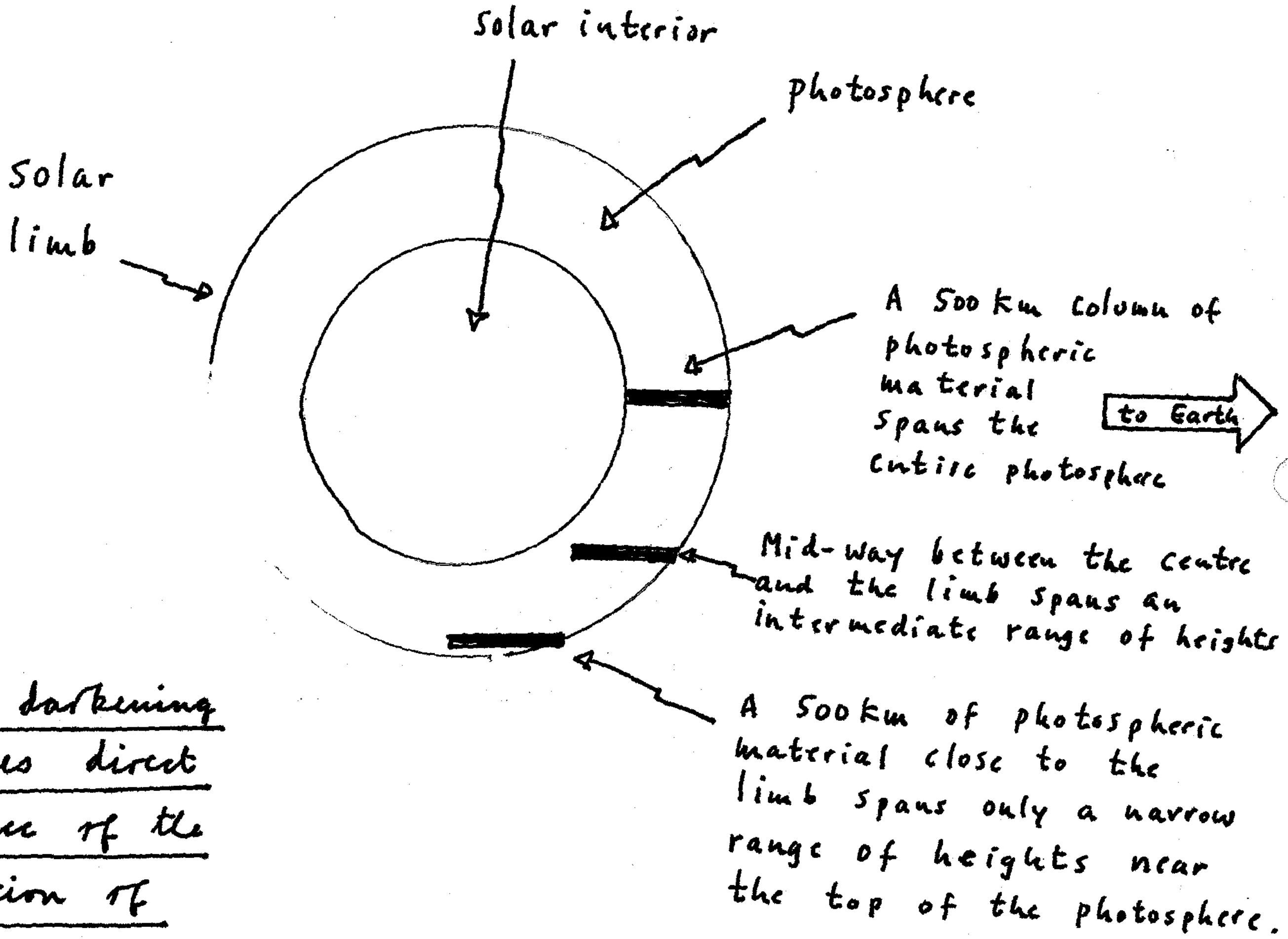
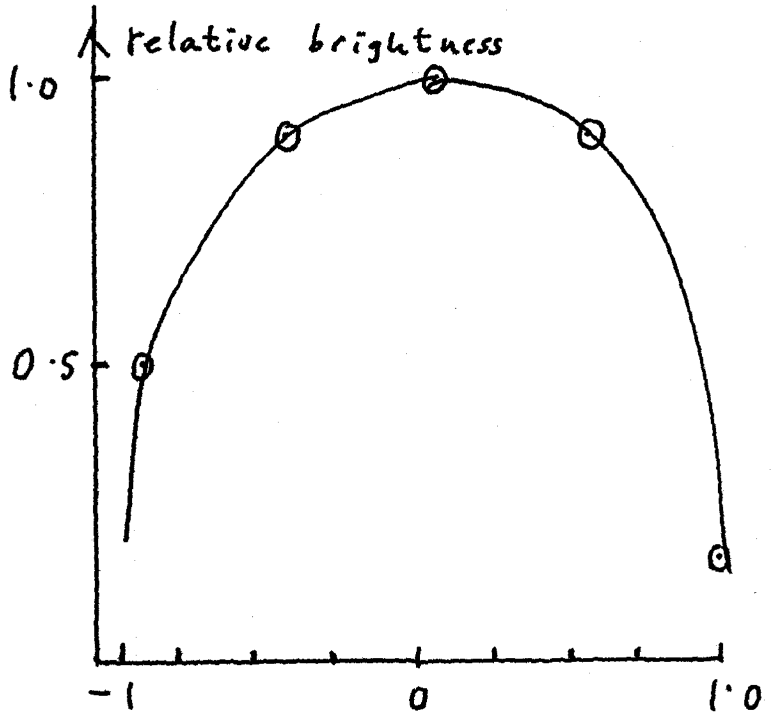
Darkening of the Solar Limb (s)



Limb darkening provides direct evidence of the variation of photospheric temperature with height.



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