Orbital and rotational kinetic energies; rotational and orbital angular momenta

DF (i)

① The Earth is an approximate sphere of radius 6×10^{6} m and moment of inertia 8×10^{37} kg m^2. Every day an estimated 1×10^{7} kg of meteoric material reaches the Earth from space, most of it in the form of extremely fine dust grains. The material arrives in approximately equal quantities from all directions.

(a) Explain the reason that the arrival of this material causes the rate of rotation of the Earth to decrease.

(b) The present mass of the Earth is 6×10^{24} kg. Calculate the fractional increase in its mass over a time interval of one million years.

② The mass of the Sun is 2.0×10^{30} kg; its rotational angular speed is 2.9×10^{-6} rad. s^{-1}. The corresponding values for Jupiter are 1.9×10^{27} kg and the orbital angular speed is 1.3×10^{4} m s^{-1}. The radius of the Jovian orbit is 7.8×10^{11} m. The Solar radius is 6.8×10^{8} m. Calculate:

(a) $L_{orbital}$ of Jupiter $= m_J \, v_J \times$ radius of its orbit

(b) $L_{rotation}$ of the Sun, regarding it as a uniform sphere of moment of inertia $\frac{2}{5} . M_{\odot} . R_{\odot}^2$

(c) The ratio of the orbital angular momentum of Jupiter to that of the rotational of the Sun.

(d) The ratio of the mass of Jupiter to that of the Solar mass.

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Reminders:

$(k.e.)_{rotational} = \frac{1}{2} . I \omega^2$

$(k.e.)_{orbital} = \frac{1}{2} m . v^2 = \frac{1}{2} m . r^2 \omega^2$

$L_{orbital} = m v . r = m . r^2 \omega$

$L_{rotational} = I \omega$

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(e) If Jupiter was formed from material ejected from the Sun, the Sun must have possessed all the angular momentum which Jupiter now has. What would its (rotational) angular speed have been before the ejection of Jupiter? (Actually, the Sun would have rotated nearly twice as quickly as this, due to the angular momentum possessed by the other planets)

Hint:

Original Mass of the Sun $= \left(6 \times 10^{30}\,\text{kg} + 1.9 \times 10^{27}\,\text{kg}\right)$

and $\left[\omega_{\text{origial}}\right]_{\odot} = \,?$

Use $I_1 \omega_1 = I_2 \omega_2$

$\therefore \omega_1 = \dfrac{I_2 \omega_2}{I_1}$

Where I_1 and I_2 are the original and present rotational angular momenta of the Sun.

(f) If Jupiter was initially flung from the Solar equator when it was spinning this rapidly, how much orbital angular momentum did the planet then have?

This underlines the problem that faces all theories of the origin of the Solar System: how do the planets with around 0.1% of the total mass of the System possess nearly 98% of the total angular momentum? About half of the stars, of which the angular rotation speeds have been measured, rotate (roughly) at the same rate as the Sun, whilst the remainder rotate much more rapidly.

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