Satellite motion

DF (i)

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

The expression for centripetal force, where M is the mass of the primary, around which the secondary is revolving.

Cancelling,

$$v^2 = \frac{GM}{r}$$

We see that the smaller the radius of the orbit, r, the more quickly the Satellite must travel.

Alternatively, we can use the other expression for centripetal force,

$$\underline{F = m r \omega^2}$$

where ω is the angular velocity (in $\underline{\text{radians } s^{-1}}$)

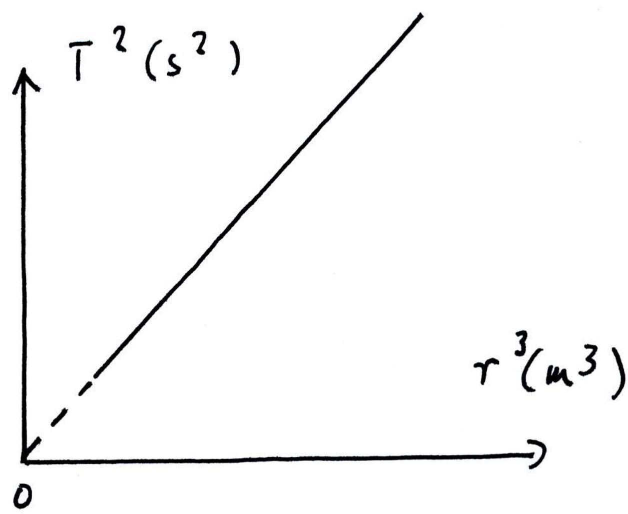
Putting this with the expression for gravitational force,

$$\frac{GMm}{r^2} = m r \omega^2 ,$$

$$\underset{\left(\frac{4\pi^2}{T^2}\right)}{\omega^2} = \frac{GM}{r^3} .$$

Since $\omega = \frac{2\pi}{T}$, where T is the period of revolution of the Satellite,

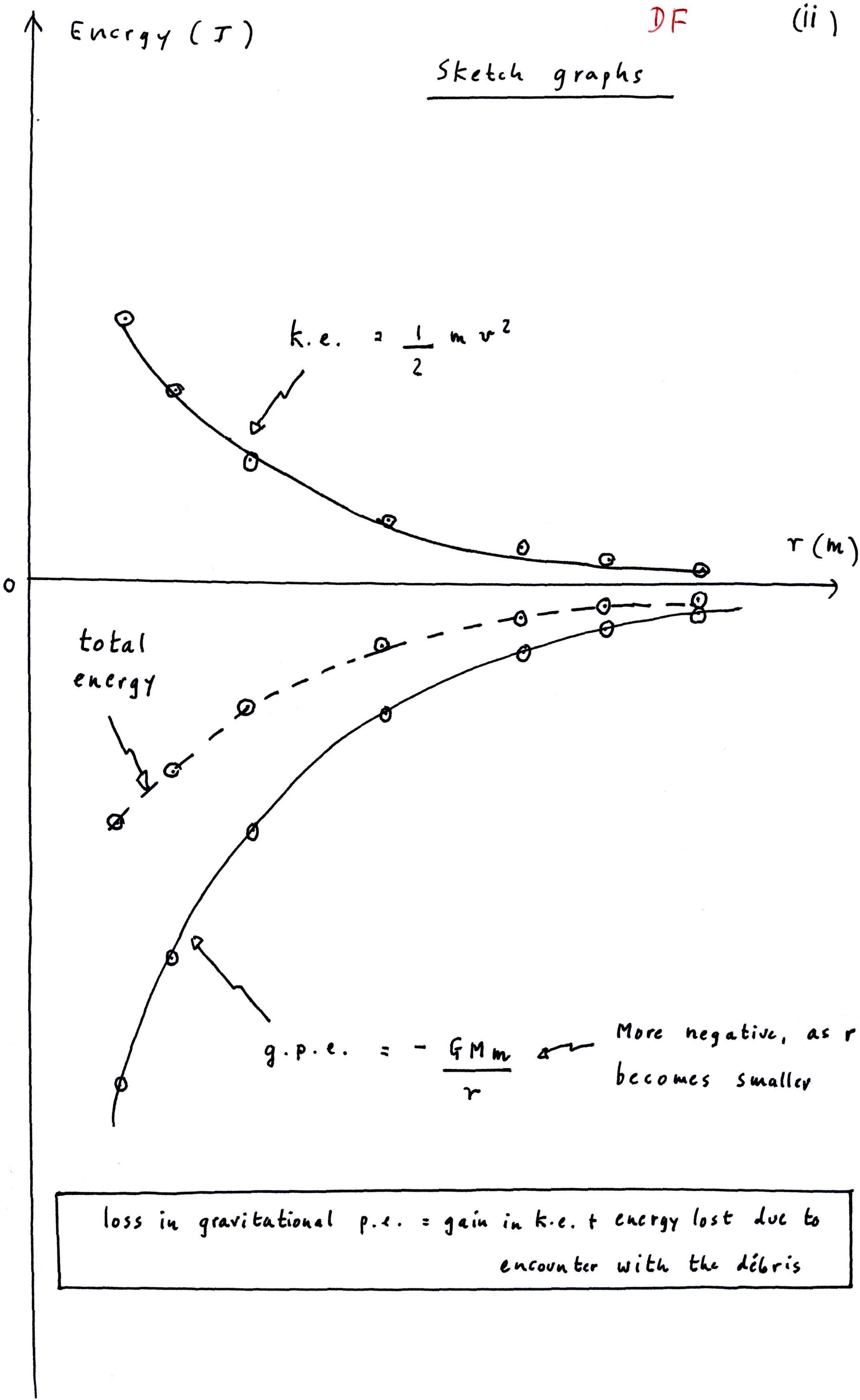
$$\underline{\underline{T^2 \propto r^3}}$$



In this case, the constant, in $k\underline{\text{III}}$, is

$$\frac{4\pi^2}{GM}$$

Sketch graphs



loss in gravitational p.e. = gain in k.e. + energy lost due to encounter with the débris

We now come to the point of this analysis. Suppose that an artificial Earth satellite is in a stable orbit, circling with little loss of kinetic energy. Imagine that it meets some débris, far from the Earth. This will slow the satellite, and it will fall into a lower orbit. In this lower orbit it has to travel more quickly. How can slowing it increase its speed? This is called the "satellite paradox".

The answer is found from the law of conservation of energy. When the satellite falls into a lower orbit, it is nearer to the Earth, it has lost gravitational p.e. (or, as in the sketch, it has less negative p.e.). In fact, it can be shown that the loss in g.p.e. is twice the gain in k.e. So, when the satellite drops into a lower orbit it has lost energy, and this a more stable state. The amount of the gravitational p.e. is given by $\frac{GMm}{r}$; from the first equation on p.(i), and re-arranging $\frac{GMm}{r} = mv^2$. This is <u>twice</u> the k.e. We can put the transaction in the form of an equation

$$\boxed{\text{loss in gravitational p.e.} = \text{gain in k.e.} + \text{energy lost due to the débris}}$$

The sketch graphs on p.(ii) show the gravitational p.e., which is negative, because work must be done to lift the satellite from the surface of the Earth (or, more correctly, from the centre of the Earth). The k.e. must <u>always</u> be positive, because the speed — or velocity — is squared. The <u>total</u> energy is still negative, because the value of the g.p.e. is <u>twice</u> that of the k.e.

DF

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