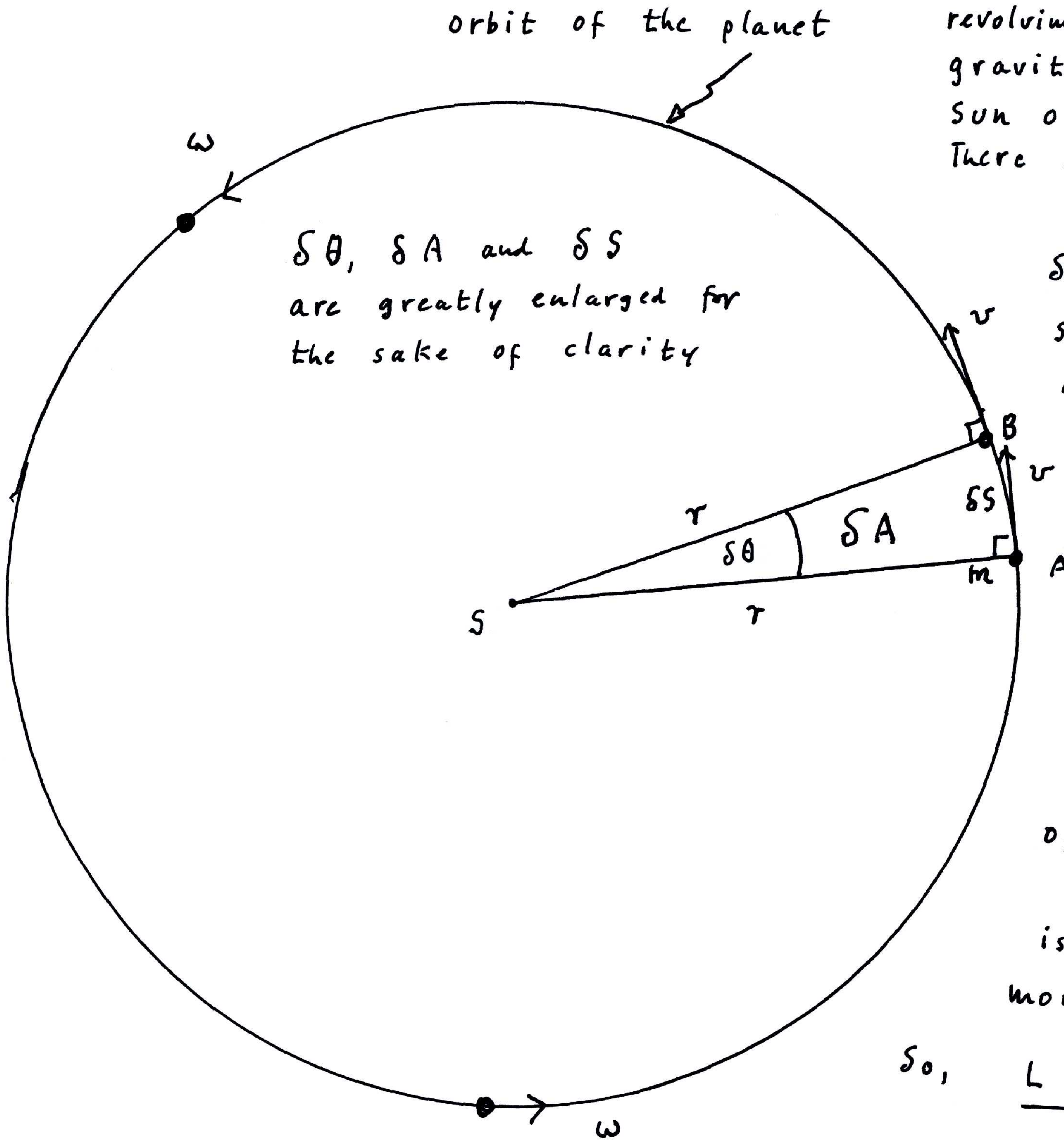
DF DF
2018, February 7

A more detailed look at Johannes Kepler's Second Law of Planetary Motion (1618)



Consider a planet of mass m, revolving around the Sun. The gravitational force, exerted by the Sun on the planet, is wholly radial. There is no transverse component.

During a time interval of δt let the radius vector sweep out an area of δA, as the planet moves a distance of δS, from A to B.

At any instant, the tangential speed of v is about the axis of revolution. That is, the Sun.

The linear momentum, $\vec{P}$, of the planet is mv.

The angular momentum, L, is the moment of the linear momentum.

So, $L = mv \times r$ ——— ①

This must be constant, because the force acting on the planet is always perpendicular to the instantaneous direction of the planet.

Refer to equation ①:

Replace v by $\dfrac{\delta S}{\delta t}$

That is, $\boxed{m \cdot \left(\dfrac{\delta S}{\delta t}\right) r = \text{Constant}}$ ——— ②

Look at the triangle ABS.

ABS is the base of this triangle, δS.

Now, base $\times$ r = twice the area swept out by the radius vector, during a time interval of δt.

Substituting into equation ②:

$$m \times \frac{2\,\delta A}{\delta t} = \text{Constant} \qquad ③$$

Rearranging equation ③.

$$\boxed{\frac{\delta A}{\delta t} = \frac{\text{constant}}{2m}}$$

Johannes Kepler (1571-1630): twenty-five years younger than Brahe and seven years younger than Galileo.

kI describes the shapes of the orbits; kII the varying speed of motion in an orbit, and the third the relation to the size of the orbit to the time of revolution. Kepler's formulation was empirical: his laws describe the orbital motions of the planets perfectly, but they do not account for them — that was left to Newton.