Ellipse parameters

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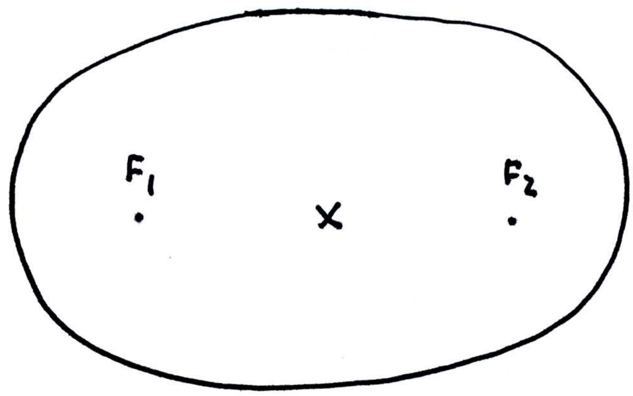
You already have a diagram illustrating the basic parameters of an ellipse. Remember that an ellipse is defined by two points, called the "foci" of the ellipse (each one is called a "focus"). For all points on the ellipse, the combined distances from both foci have the same value. This is the reason that you were able to draw your ellipse by inserting a drawing-pin at each focus, attaching a string (loosely) between them, and stretching the string tightly with the tip of your pen and moving all the way around both foci.

The properties of ellipses are important for several reasons. For example, all the permanent members of the Sun revolve in elliptical orbits of different eccentricities; the motion of the Moon around the Earth is elliptical, with the perigee considerably less than the apogee.

As long as you do not put your foci on top of each other, the resulting will have a (relatively) long Major Axis and a shorter Minor Axis, with the length of the string equal to the Major Axis. Half the Major Axis is called the semi-Major Axis (denoted by "a"); half the Minor Axis is called the semi-Minor Axis (denoted by "b"). The distance from the centre of the ellipse (marked with an "X")



to each focus is sometimes called "f", so the distance between the two foci is equal to 2f.

If you study the equation defining the eccentricity, e,

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Where b is the semi-Minor Axis and a is the semi-Major Axis,

it is clear that, for a circle (for which $a = b$) has an eccentricity of zero. Referring to the equation, when $a = b$, $\frac{b^2}{a^2} = 1$, and $\sqrt{1 - 1} =$ zero

For a highly-flattened ellipse ("a" <u>much</u> greater than "b") $\frac{b^2}{a^2}$ is small. That is, $1 - \frac{b^2}{a^2} \approx 1$.

Suppose, for example, $\frac{b}{a} = \frac{1}{10}$. Then $\frac{b^2}{a^2} = \frac{1}{100}$

Substituting into the equation,

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \frac{1}{100}}$$

$$= \sqrt{0 \cdot 99}$$

$$\underline{= 0 \cdot 995}$$

Consider a much less flattened ellipse for which $\frac{b}{a} = \frac{3}{4}$

$$\therefore \frac{b^2}{a^2} = \frac{9}{16}$$

$$= 0 \cdot 56$$

$$\therefore e = \sqrt{1 - 0 \cdot 56}$$

$$= \sqrt{0 \cdot 44}$$

$\therefore$ <u>"e" is calculated to be 0·66, for an orbit which has these parameters</u>

For my drawing of an ellipse, on page 4,

$b = 9 \cdot 9$ cm and

$a = 12 \cdot 2$ cm

$$\frac{b}{a} = \frac{9 \cdot 9 \text{ cm}}{12 \cdot 2 \text{ cm}}$$

$$= 0 \cdot 81$$

$$\therefore \frac{b^2}{a^2} = (0 \cdot 81)^2$$

$$\underline{= 0 \cdot 65(5)}$$

Using $e = \sqrt{1 - \frac{b^2}{a^2}}$

and substituting :

$$e = \sqrt{1 - 0 \cdot 65(5)}$$

$$= \sqrt{0 \cdot 35}$$

$$\underline{= 0 \cdot 59}$$

<u>The ellipticity is calculated to be 0·59.</u>

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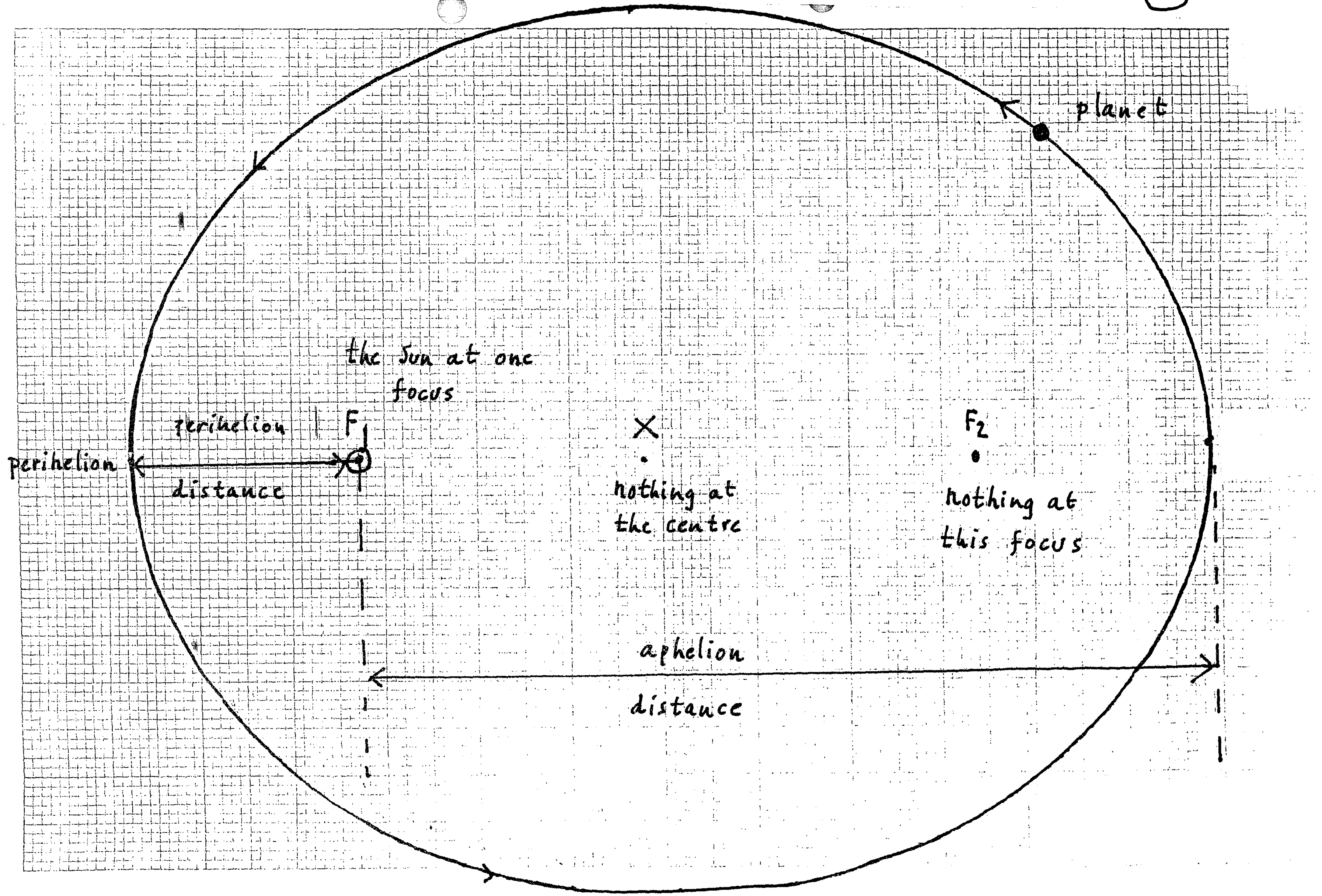
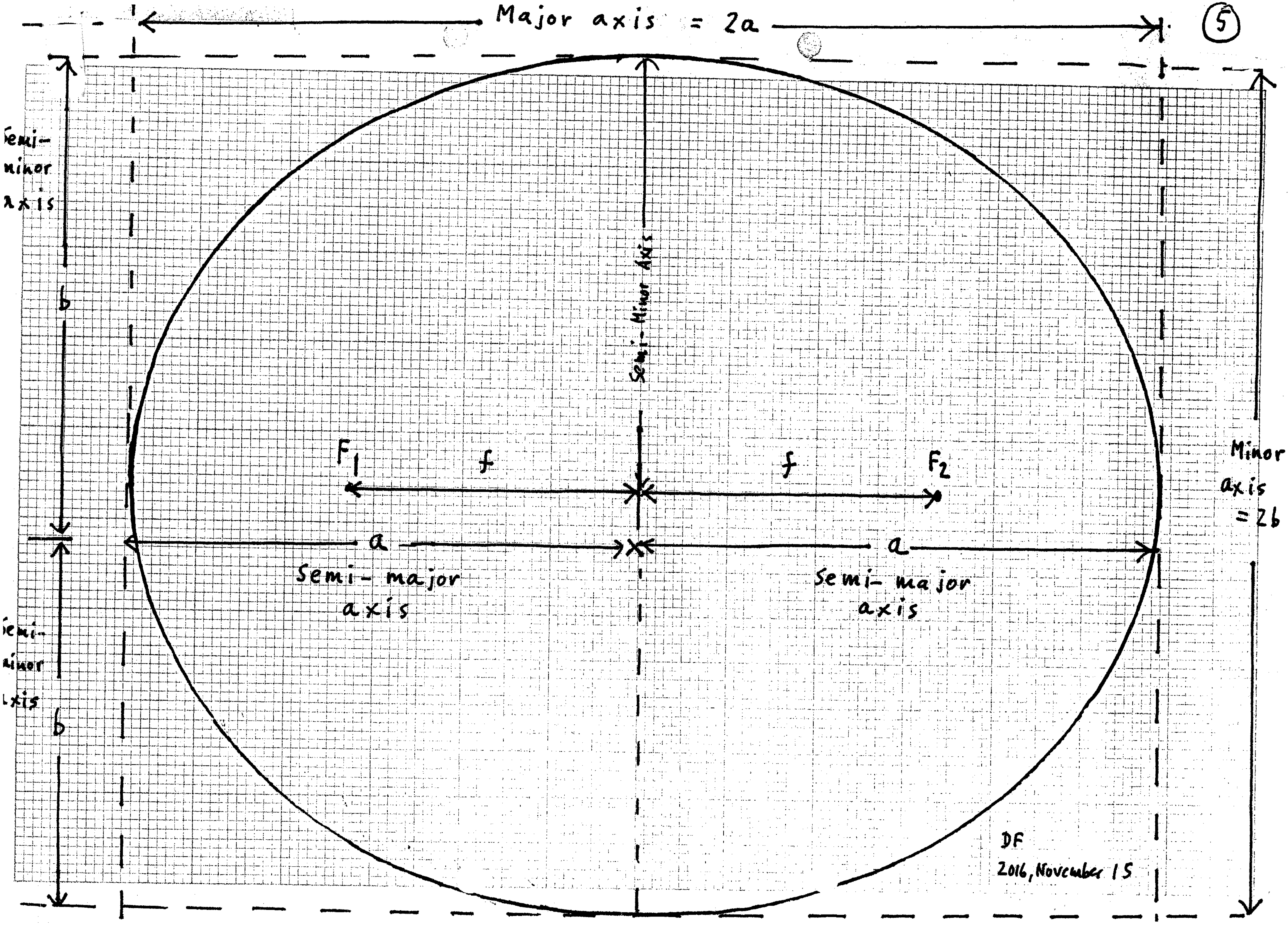
Parameters of an elliptical orbit




Major axis = 2a
Semi-minor axis
b
Semi-Minor Axis
F1
f
f
F2
a
a
Semi-major axis
Semi-major axis
Semi-minor axis
b
Minor axis = 2b
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