A.7 (a) answers

① (a) α Centauri is 1·3 pc away from the Earth, i.e., $1\cdot3 \times 2\times10^{5}$ A.U.
At a distance of 1 A.U. the apparent magnitude of the Sun is −27 and Jupiter would be −5.

According to the inverse-square law relating distance and brightness at 1·3 pc the brightness of the Sun would be $\left(\dfrac{1}{1\cdot3\times2\times10^{5}}\right)^{2}$ as bright as it is at a distance of 1 A.U.
i.e., $1\cdot48\times10^{-11}$ times as bright.

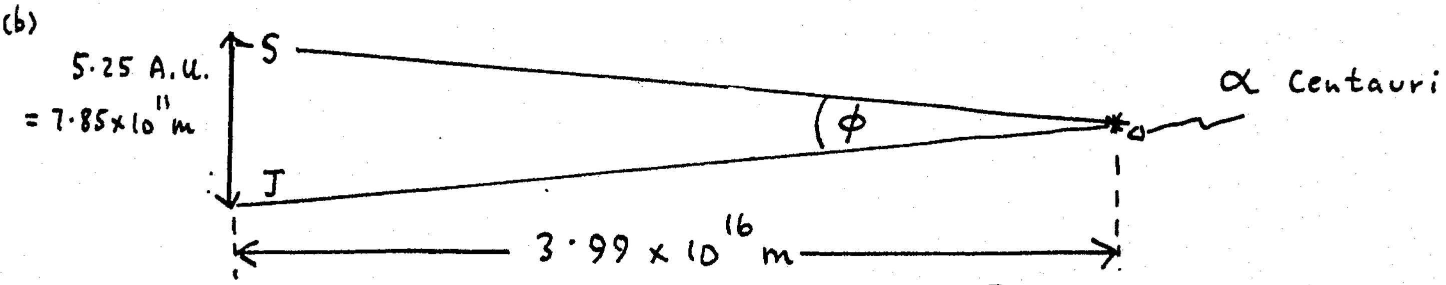
However, according to the Pogson equation if the brightness is decreasing by a certain amount, the magnitude can be related
i.e. $(1\cdot3\times2\times10^{5})^{2} = (2\cdot5)^{x}$, where x is the magnitude difference
Taking logarithms to both sides

$$\log_{10}(1\cdot3\times2\times10^{5})^{2} = x\log_{10}2\cdot5 \quad \text{and} \quad \frac{\log_{10}(1\cdot3\times2\times10^{5})^{2}}{\log_{10}2\cdot5} = x$$

$\therefore\ x = 27\cdot22$

Thus, in both the cases of the Sun and Jupiter, the difference in apparent magnitude is 27·22 [this should be "added" to the previous figures, because both are now fainter and the greater the negative value the brighter is the object]
Thus, the apparent magnitudes of the Sun and Jupiter as seen from α Centauri are +0·22 + 22·22, respectively.

(b)



in radians, $\phi = \dfrac{7\cdot85\times10^{11}\,\text{m}}{3\cdot99\times10^{16}\,\text{m}} = 1\cdot96\times10^{-5}$

The sine of $\left(\dfrac{\phi}{2}\right) = \dfrac{3\cdot93\times10^{11}\,\text{m}}{3\cdot99\times10^{16}\,\text{m}} = 9\cdot85\times10^{-6}$ i.e. $\phi = 1\cdot13\times10^{-3\,\circ}$

② 1 pc = 2×10^{5} A.U. 10 pc = 2×10^{6} A.U.
The brightness at 1 A.U. = $(2\times10^{6})^{2}$ than the brightness would be at 10 pc $\therefore$ allowing x to be the magnitude difference between the two $(2\times10^{6})^{2} = (2\cdot5)^{x}$

Taking logarithms to both sides

$$\log_{10}(2\times10^{6})^{2} = x\log_{10}2\cdot5$$

i.e., $$x = \frac{\log_{10}(2\times10^{6})^{2}}{\log_{10}2\cdot5}$$

$x = 31\cdot7$

This is the magnitude difference between that of the Sun at its present distance and its absolute magnitude, that is, at 10 pc

The apparent magnitude of the Sun is -27 (at 1 A.U.) Its absolute magnitude would be $27 + 31 \cdot 7 \approx 5$.

Let us assume that each of the 10^{11} stars resembles the Sun (in luminosity). Some will be more / less luminous. Presumably, this will cause the brightness to increase by 10^{11}.

Thus, in order to calculate the absolute magnitude of the Galaxy, we presume

$10^{11} = (2 \cdot 5)^x$, where x is the magnitude difference between one Sun and 10^{11} suns.

$$\therefore \quad \log_{10} 10^{11} = x \log_{10} 2 \cdot 5$$

$$\therefore \quad \frac{\log_{10} 10^{11}}{\log_{10} 2 \cdot 5} = x \qquad \underline{\text{i.e., } x = 27 \cdot 6}$$

Since we expect 10^{11} suns to have a greater magnitude than one Sun; the new magnitude should be negative, that is, $5 - 27 \cdot 6 \approx -23$

<u>The total absolute magnitude of the Galaxy is approximately -23</u>

③ Distance of the Virgo cluster of galaxies is 8×10^6 pc

Since $1 \text{ pc} = 3 \cdot 086 \times 10^{16}$ m

$$\therefore \quad 8 \times 10^6 \text{ pc} = 8 \times 10^6 \text{ pc} \times 3 \cdot 086 \times 10^{16} \text{ m (pc)}^{-1}$$

$$= \underline{2 \cdot 47 \times 10^{23} \text{ m}}.$$

Light travels at $3 \times 10^8 \text{ m s}^{-1}$ $\therefore$ the light from this cluster will take $\dfrac{2 \cdot 47 \times 10^{23} \text{ m}}{3 \times 10^8 \text{ m s}^{-1}}$

$= \underline{8 \cdot 23 \times 10^{14} \text{ s}}$ to reach the astronomer

To convert this figure into years:

$$\frac{8 \cdot 23 \times 10^{14} \text{ s}}{60 \times 60 \times 24 \times 365 \text{ seconds (year)}^{-1}}$$

$= 26\ 100\ 000$ years.

<u>The astronomer would see the group as it existed 26 100 000 years ago (sic)</u>

cf. the Sun : eight <u>light minutes</u>
the Moon : about 1·25 <u>light seconds</u>

The diameter of the Solar System : about five <u>light hours</u>

Absolute magnitude, Apparent magnitude and Distance

Let L be the amount of energy which would be received from the star per m^2 per second if it were at a distance of ten pc and M its absolute magnitude. Let ℓ and m be the corresponding apparent surface brightness and magnitude at its true distance of (say) d parsecs. Then, since radiant energy obeys the inverse - square law,

$$\frac{L}{\ell} = \frac{d^2}{100}.$$

Also, since a difference of five magnitudes corresponds to a factor of one hundred in brightness (see A.3), then, a difference of x magnitudes corresponds to a factor of $100^{x/5}$

and

$$\frac{L}{\ell} = 100^{(m-M)/5}$$

thus, $\frac{d^2}{100} = 100^{(m-M)/5}$.

Taking logarithms to base 10:

$$2\log d - 2\log 10 = 2(m-M)/5$$

or $\boxed{M = m + 5 - 5\log d}$

Thus,

$$\log d = \frac{m-M}{5} + 1$$

For example, Rigel (α Orionis) has an apparent magnitude $+0\cdot3$ and an absolute magnitude $-5\cdot8$. Substituting in the above gives

$$\log d = \frac{0\cdot3 + 5\cdot8}{5} + 1 = 2\cdot2 \quad \therefore d = 160\,\text{pc}$$

For many spectral classes it was discovered that the ratio of the intensities certain pairs of lines was related to the absolute magnitude of the star. $\therefore$ absolute magnitude is inferred from spectral class. Such parallaxes are called <u>spectroscopic parallaxes</u>.