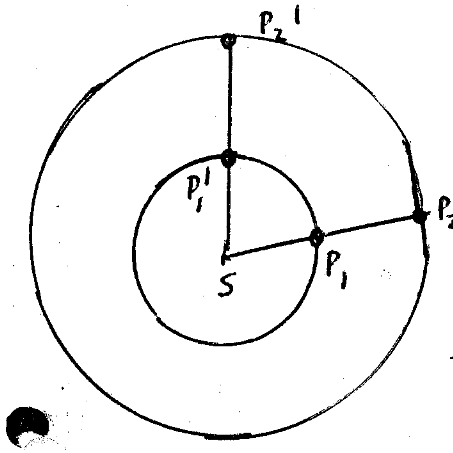
Another look at the Synodic Period ...



Let T_1 and T_2 be the sideral periods of revolution of two planets P_1 and P_2 about the Sun. Assume that the planets move in circular, coplanar orbits. In such an orbit, the radius vector sweeps out equal angles in equal times, i.e., the angular velocity of the planet is constant. Let these be n_1 and n_2, respectively. Then,

$$n_1 = \frac{360^\circ}{T_1} \; ; \qquad n_2 = \frac{360^\circ}{T_2} \qquad \text{(1)}$$

Now, the planet nearer the Sun has a smaller period of revolution than the one farther away, so that $T_1 < T_2$. Hence $n_1 > n_2$ and the radius vector SP_1 therefore gains on the radius vector SP_2 by $(n_1 - n_2)$ degrees per day.

Let the configuration SP_1P_2 denote the positions of the Sun and the two planets at a particular epoch. Then, without at the moment specifying which planet is the Earth, it is clear that a synodic period, (say) S, will have elapsed by the time the next similar configuration $SP_1^1P_2^1$ occurs. During the time interval, S, the radius vector of P_1 will have gained 360° on the radius vector of P_2

But SP_1 gains on SP_2 by $(n_1 - n_2)$ degrees per day, so that it gains 360° in a time S, where $S \times (n_1 - n_2) = 360^\circ$, or, using relations (1)

$$S\left(\frac{360^\circ}{T_1} - \frac{360^\circ}{T_2}\right) = 360^\circ$$

giving

$$\boxed{\frac{1}{S} = \frac{1}{T_1} - \frac{1}{T_2}} \qquad \text{(2)}$$

Case (a): If the planet is inferior, T_1 refers to the sideral period of the planet; T_2 refers to the Earth's.

Case (b): If the planet is superior, T_1 refer to the Earth's sideral period; T_2 to that of the planet.

The synodic period of Venus was found to be 583·9 days. If the length of the year is 365·25 days, calculate the sidereal period of Venus. Since Venus is an inferior planet,

$$\frac{1}{583{\cdot}9} = \frac{1}{T_1} - \frac{1}{365{\cdot}25}$$

giving $\underline{T_1 = 224{\cdot}7 \text{ days}}$